Federal University, Oye-Ekiti, Ekiti State Nigeria.
Department of Civil Engineering
CVE 204 – Strength of Materials

Shear Force and Bending Moment Diagram of Determinate Beams

1.0 General Background

1.1 Beam – structural member that carries load normal or perpendicular to its longitudinal axis

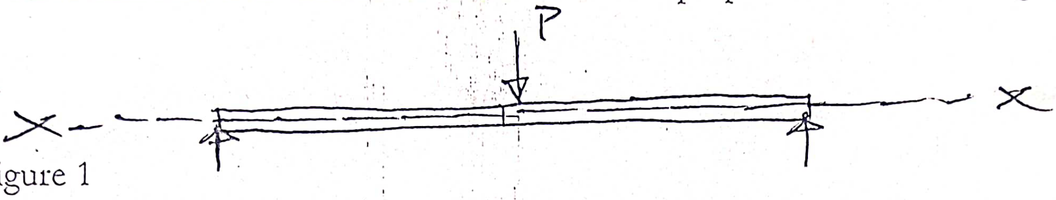


Figure 1

The action of the normal loads leads to the development of two types of response named (i) shear force and (ii) bending moments, at the cross section of the beam.

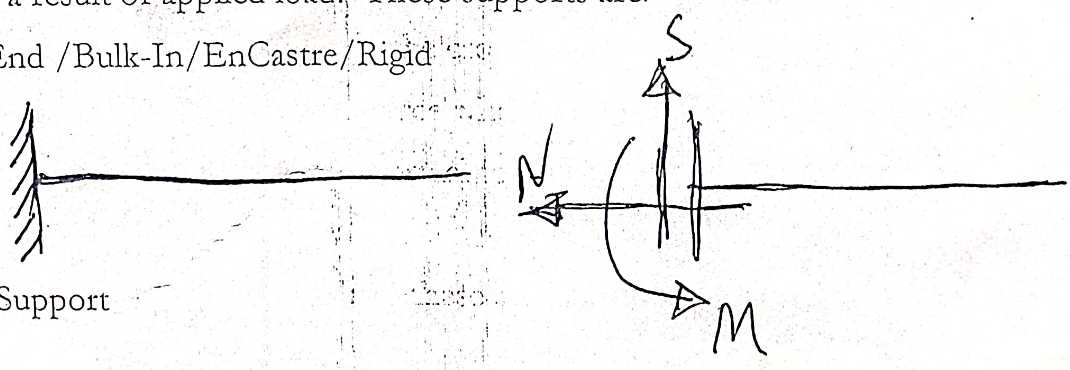
Unlike when beam is subjected to axial force where tensile and compressive stresses are developed, subjecting a beam to a load normal to its longitudinal axis led to the development of shear force and bending moments.

We shall know how to calculate and draw these stresses.

1.2 Types of Support in a Beam Structure

The supports in a beam supply the necessary reactive forces to maintain the structure in equilibrium as a result of applied load. These supports are:

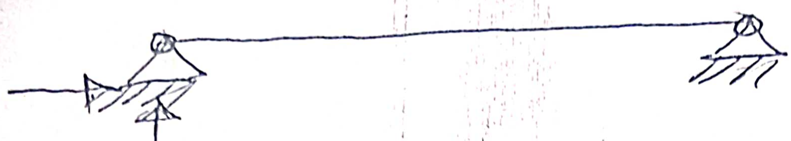
a) Fixed End /Bulk-In/EnCastre/Rigid



Fixed Support

The support is capable of supplying three reactive forces: horizontal, vertical, and a fixing moment. This type of support is fixed so that it cannot rotate under the action of superimposes load.

Pin (or Hinged) Support

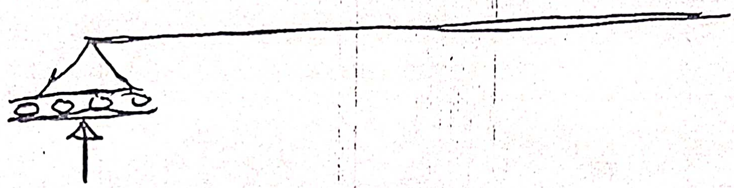


Pinned Support

This is assumed to be free to rotate under the applied load, and can support two reactions. These are: horizontal and vertical forces.

c) The Roller Bearing

This can only supply one reaction. This is the case when a beam simply rests on a support



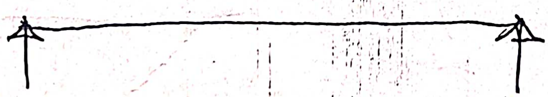
Roller Bearing Support

1.3 Types of Beams

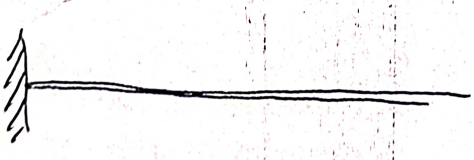
Beams can be classified in many ways

i. Classification based on Support system:

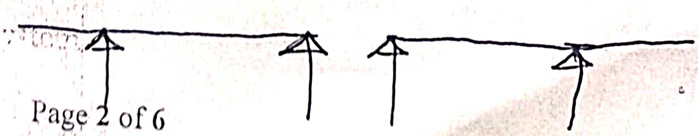
a) Simply supported



b) Cantilever

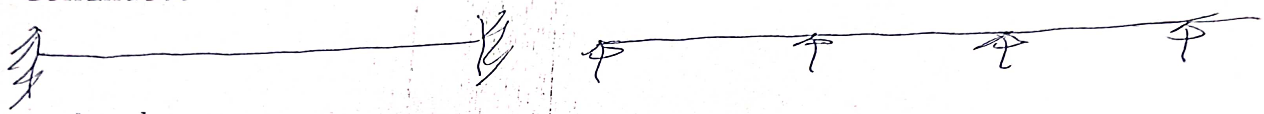


c) Simply supported with overhangs



d) Propped cantilever

e) Continuous



f) Fixed ends

ii. Classifications based of Method of Analysis

a) Determinate – can be analysed by the eqn of static equilibrium

For 2-dimensional beam structure

$\Sigma F_y = 0$

$\Sigma F_x = 0$

$M_z = 0$

For 3-dimesnsional beam structure

$\Sigma F_x = 0$ $\qquad \Sigma M_x = 0$

$\Sigma F_y = 0$ $\qquad \Sigma M_y = 0$

$\Sigma F_z = 0$ $\qquad \Sigma M_z = 0$

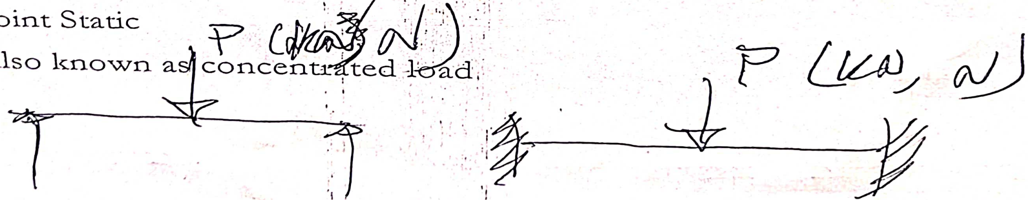
b) Indeterminate

can not be analysed by the application of static equilibrium

1.4 Types of Loads

Many forms of loads are dealt with in structural analysis. They include:

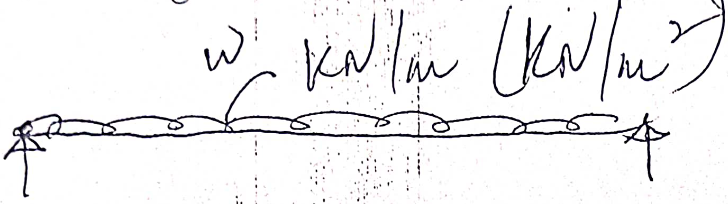
1. Static Load
   i) Point Static
      Also known as concentrated load.



ii) Distributed Load (udl)

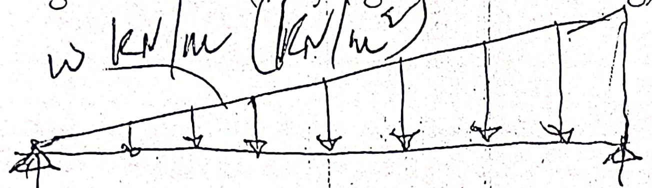
This is when a load is applied in such a way that it spread over the entire or part of its length.

a. When the loading rate is uniform, it is called uniformly distributed load.

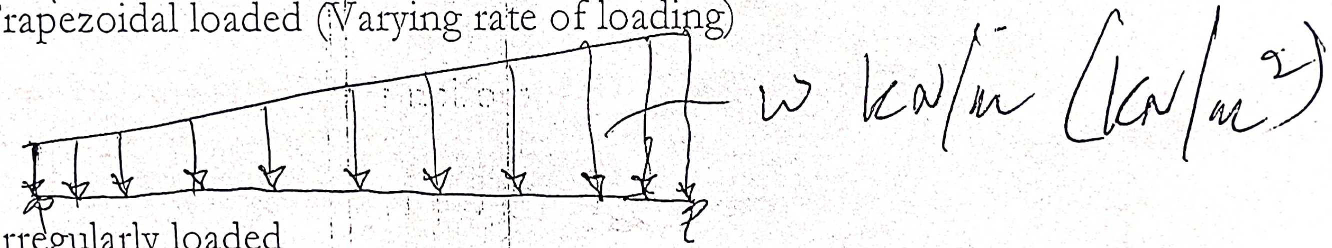


Uniformly distributed load (constant rate of loading

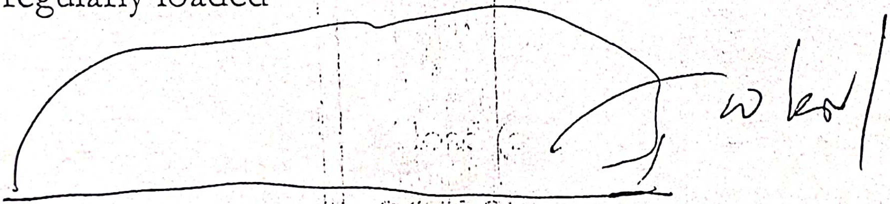
b. Triangular Load (varying rate of loading)



c. Trapezoidal loaded (Varying rate of loading)

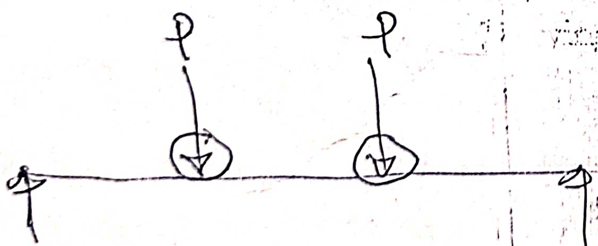


d. Irregularly loaded



2. Dynamic Loads

These are moving loads. It usually occurs in highway/bridge analysis and design.

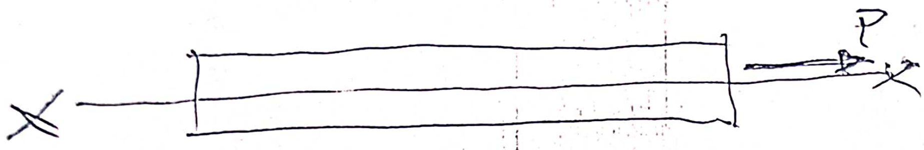


SHEAR FORCE AND BENDING MOMENTS

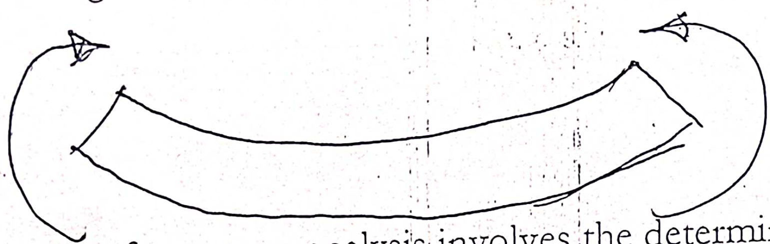
forces acting in a structural member may include some or all the following:

a. Forces normal to elemental axis e.g. shear force.

b. Forces parallel to elemental axis e.g. Axial or normal forces



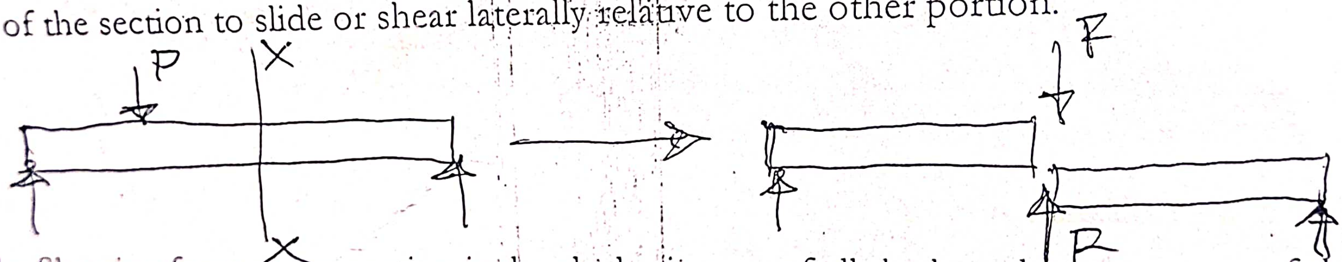
c. Bending Actions. For example, bending moment and twisting moment



In the beam type of structure, analysis involves the determination of the magnitude of the Shear force (SF) and the Bending moment (BM) at all points of the beam.

The Shearing Force

The shearing Force at any section of a beam is the tendency of the portion of the beam on one side of the section to slide or shear laterally relative to the other portion.



The Shearing force at any section is the algebraic sum of all the lateral components of the forces acting on either side of the section.

The shearing force diagram is the one which shows the variation of shearing force along the length of the beam

Shearing force is +ve when the resultant to the left is upward, and –ve when resultant to the right is downward

Bending Moment

bending moment about any section is the algebraic sum of all the moments about that ction of all the forces acting on either side of the section

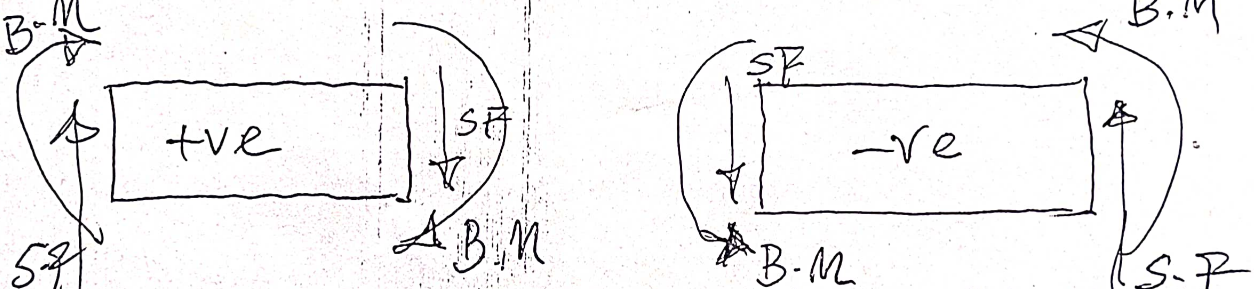
The shearing force diagram is the one which shows the variation of shearing force along the length of the beam

Bending moment can be of two types:

a. Sagging moment → +ve

b. Hogging moment → -ve

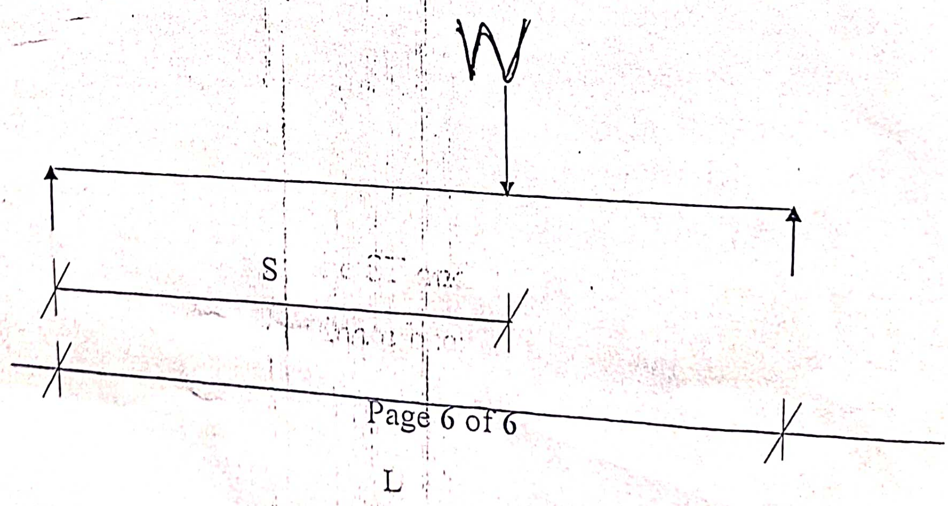
Overall Diagram



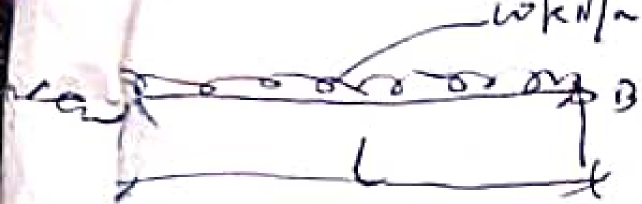
The following examples will show how the SF and BM are calculated as well show how SF and BM diagrams are constructed for determinate beams.

Example 1

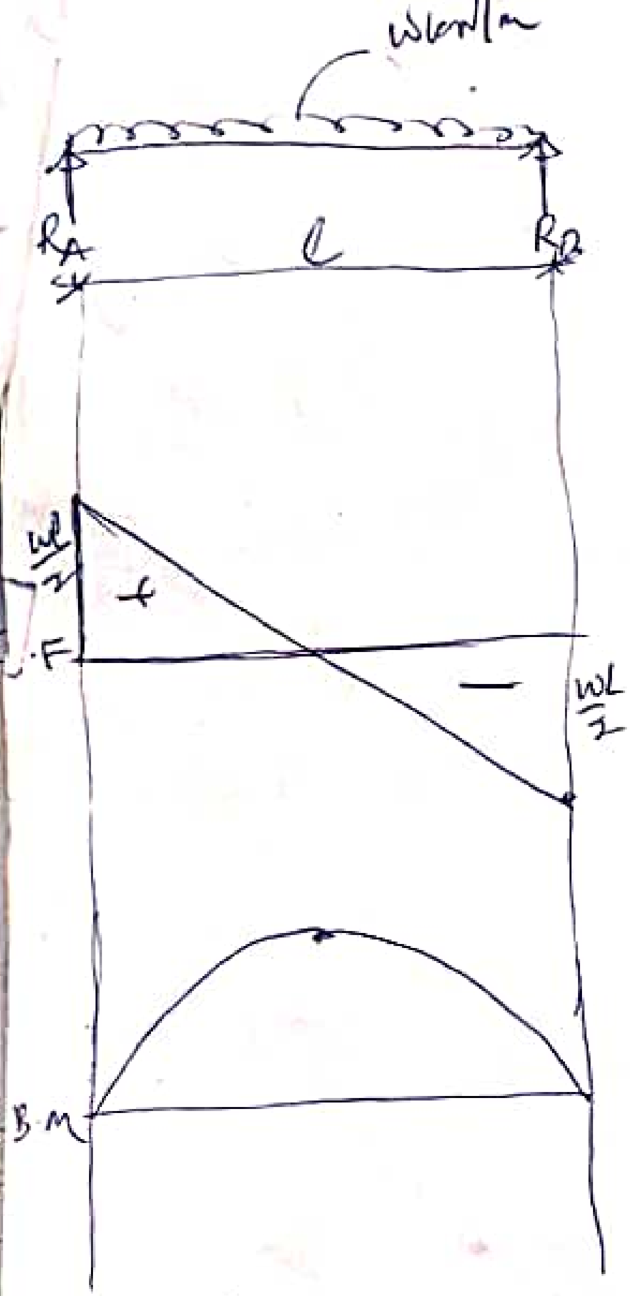
Draw the shear force and bending moment diagram for a simply supported beam with a point load as shown



Example 2



Solution



Total load = wL kN

To find R_A & R_B

$\Sigma F_y = 0$

$R_A + R_B = wL$ —— (1)

ΣM_B (or A)

(3)

$-R_B L + wL \cdot \frac{L}{2} = 0$

$R_B L = \frac{wL^2}{2}$

$R_B = \frac{wL}{2}$ kN —— (2)

from eqn 1

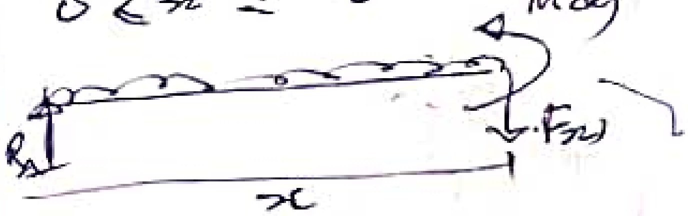
$R_A = wL - R_B$

$= wL - \frac{wL}{2}$

$= \frac{wL}{2}$ —— (3)

Just one section is necessary

$0 \leq x \leq L$



S.F

$\Sigma F_y = 0$

$R_A - wx - F_x = 0$

$F_x = R_A - wx$

$= \frac{wL}{2} - wx$ —— (4)

this is a general eqn ...

at $x = 0$

$F_{(x=0)} = \frac{wL}{2} - 0$

$= \frac{wL}{2}$ kN

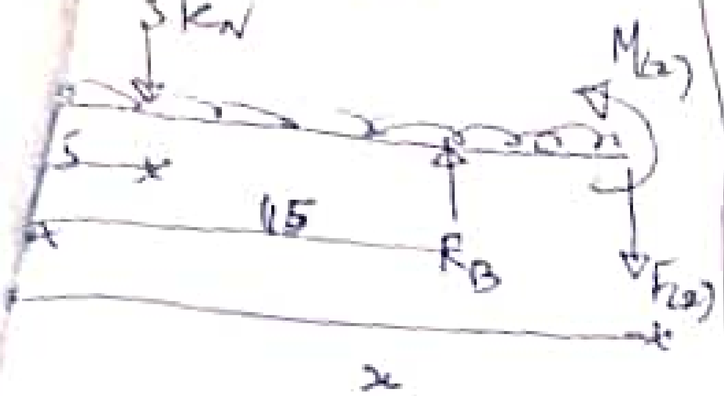
at $x = L$

$F_{(x=L)} = \frac{wL}{2} - wL$

$= -\frac{wL}{2}$ kN

3

$< x < 20$



SF..

$R_A - 5 - 2x + R_B - F(x) = 0$

$F(x) = R_A + R_B - 5 - 2x$

$= 16.67 + 28.33 - 5 - 2x$

$= 45 - 5 - 2x$

$\boxed{= 40 - 2x}$ (6)

at $x = 15$ m

$F(x) = 40 - 30$
$= 10$ kN

at $x = 20$ m

$F(x) = 40 - 40$
$= 0$ kN

BM

$R_A x - 5(x-5) - 2\cdot x\cdot \frac{x}{2} + R_B(x-15) - M(x) = 0$

$R_A x - 5x + 25 - x^2 + R_B x - 15R_B - M(x) = 0$

$x(R_A - 5 + R_B) + 25 - 15R_B - x^2 = M(x)$

$40x + 25 - 15\times 28.33 - x^2 = M($

$40x - 399.95 - x^2 = M(x)$

$M(x) = -x^2 + 40x - 399.95$

at $x = 15$ m

$M(x) = -225 + 600 - 399.95$
$= 24.95$ kN.m

at $x = 20$ m

$M(x) = -(20)^2 + 40\times 20 - 399.95$
$= -400 + 800 - 399.95$
≈ 0 kN.m

$0.1 = W$

$= 0$

$x - Wx \cdot \frac{x}{2} - M_x = 0$

$M_x = R_A x - \frac{Wx^2}{2}$

$= \frac{Wlx}{2} - \frac{Wx^2}{2}$ —— (2)

This is the B.M. expression & it is quadratic

at $x = 0$

$M_{(x=0)} = \frac{Wl(0)}{2} - \frac{W(0)^2}{2}$

$= 0 \text{ kN.m}$

at $x = L$

$M_{(x=L)} = \frac{WLl}{2} - \frac{Wl^2}{2}$

$= 0 \text{ kN.m}$

For a simply supported beam carrying a udl, the relationship between the SF & BM is that at the point of zero shear, the BM is maximum

$F = \frac{dM}{dx} = 0$

$= R_A - Wx$

$= 0$

$x = \frac{R_A}{W} = \frac{\frac{WL}{2}}{W} = \frac{L}{2} \text{ m}$ (4)

...gn. It w a
...ection in x

$= R_A \cdot \frac{W(L-S)}{L}(0)$

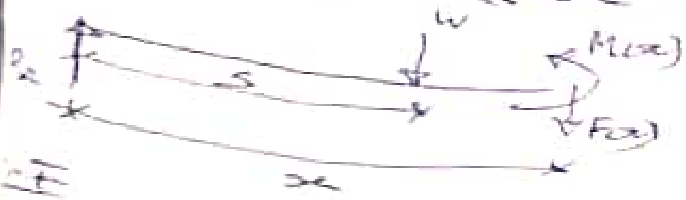
$= 0$

$x = S$

$M_{x=S} = \frac{W(L-S)S}{L}$

$= \frac{WS(L-S)}{L}$

Section $2 \to S \le x \le L$



SF

$\sum F_y = 0$

$R_A - W - F_x = 0$

$F_x = \cancel{W - R_A}\ R_A - W$

$= \frac{W(L-S)}{L} - W$

$=$

$x = -\frac{WS}{L}$

... is expression. It is the ...

$x = 0$

$F_x = -\frac{WS}{L}$

$F_{x=L} = -\frac{WS}{L}$

BM

$R_A x - W(x-S) - M_{(x)} = 0$

$M_x = R_A x - W(x-S)$

$= \frac{W(L-S)}{L}x - W(x-S)$

$= \cancel{Wx} - \frac{WSx}{L} - \cancel{Wx} + WS$

$= -\frac{WSx}{L} + WS$

This is the BM expression

at $x = S$

$M_{(x=S)} = -\frac{WSS}{L} + WS$

$= WS\left(1 - \frac{S}{L}\right)$

$= WS\frac{(L-S)}{L}$

at $x = L$

$M_{(x=L)} = -\frac{WSL}{L} + WS$

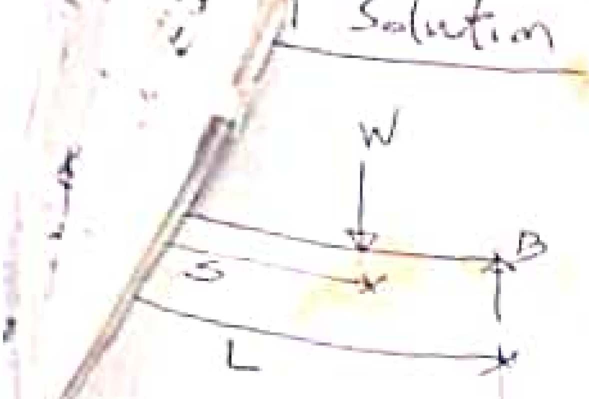
$= -WS + WS$

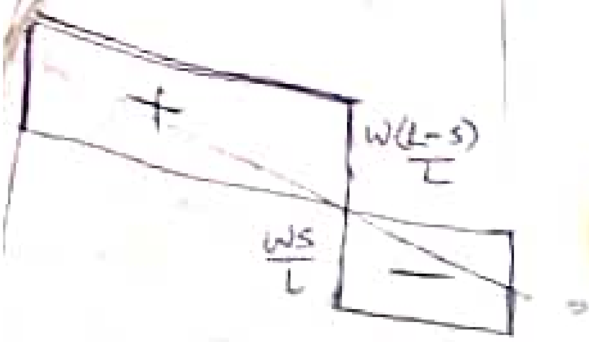
$= 0$ kN.m

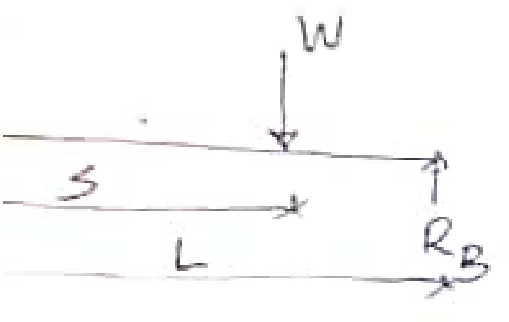
Conclusion

For simply supported beam & point load,

1 Solution







c Rxns A & B

$= 0$

$+ R_B = W$ —— (1)

$L + WS = 0$ —— (2)

$R_B L = WS$

$R_B = \frac{WS}{L}$ —— (2)

from eqn 1

$R_A = W - R_B$

$= W - \frac{WS}{L}$

$= W\left(1 - \frac{S}{L}\right)$ —— 3a

$= \frac{W(L-S)}{L}$ —— 3b

Section 1 $0 \le x \le S$

SF

$\Sigma F_y = 0$

$R_A - F_x = 0$

$F_x = R_A$

$= \frac{W(L-S)}{L}$

This is the general eqn of S.F for the section. constant w.r.t. function

at $x = 0$, $F_0 = \frac{W(L-S)}{L}$

at $x = S$ $F_S = \frac{W(L-S)}{L}$

BM

$\Sigma M_0 = 0$

$R_A x - M_x = 0$

(1) $M_x = R_A x = \frac{W(L-S)x}{L}$

$x - 2 \cdot x \cdot \frac{x}{2} - M_{(x)} = 0$

$M_{(x)} = R_A x - x^2$

$\boxed{= 16.67x - x^2}$ (3)

at $x = 0$

$M_{(x)} = 0$

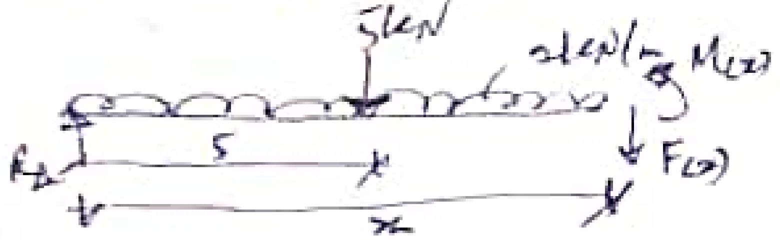
at $x = 5$ m

$M_{(x)} = 16.67 \times 5 - (5)^2$

$= 83.35 - 25$

$= 58.35$ kN.m

Section 2 $5 < x < 15$



S.F.

$R_A - 2x - 5 - F(x) = 0$

$F(x) = R_A - 2x - 5$

$= 16.67 - 2x - 5$

$\boxed{= 11.67 - 2x}$ — (4)

at $x = 5$ m

$F(x) = 11.67 - 10$

$= 1.67$ kN

$x = 15$ m

$F(x) = 11.67 - 2 \times 15$

$= 11.67 - 30$

$= -18.33$ kN

At point where S.F. = 0

eqn 4 = 0

that is

$11.67 - 2x = 0$

$2x = 11.67$

$\boxed{x = 5.84 \text{ m (from the left)}}$

<u>B.M</u>

$R_A x - 5(x-5) - 2 \cdot x \cdot \frac{x}{2} - M_{(x)} = 0$

$M_{(x)} = R_A x - 5x + 25 - x^2$

$= 16.67x - 5x + 25 - x^2$

$= 11.67x + 25 - x^2$

$\boxed{= -x^2 + 11.67x + 25}$ (5)

at $x = 5$ m

$M_{(x)} = -25 + 58.35 + 25$

$-(5)^2$

$= 58.35$ kN.m

at $x = 15$ m

$M_{(x)} = -(15)^2 + 175.05 + 25$

$= -225 + 175.05 + 25$

$= -24.95$ kN.m

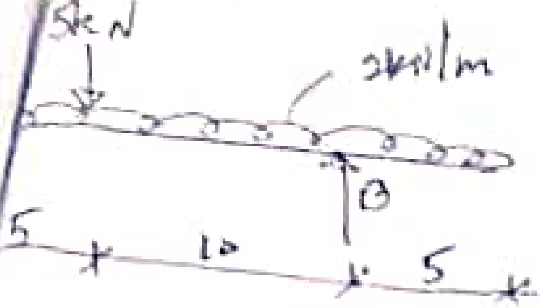
Max moment at $x = 5.84$

$M_{max} = -(5.84)^2 + 11.67 \times 5.84 + 25$

$= -34.11 + 68.15 + 25$

$\boxed{= 59.04 \text{ kN.m}}$

ple 6



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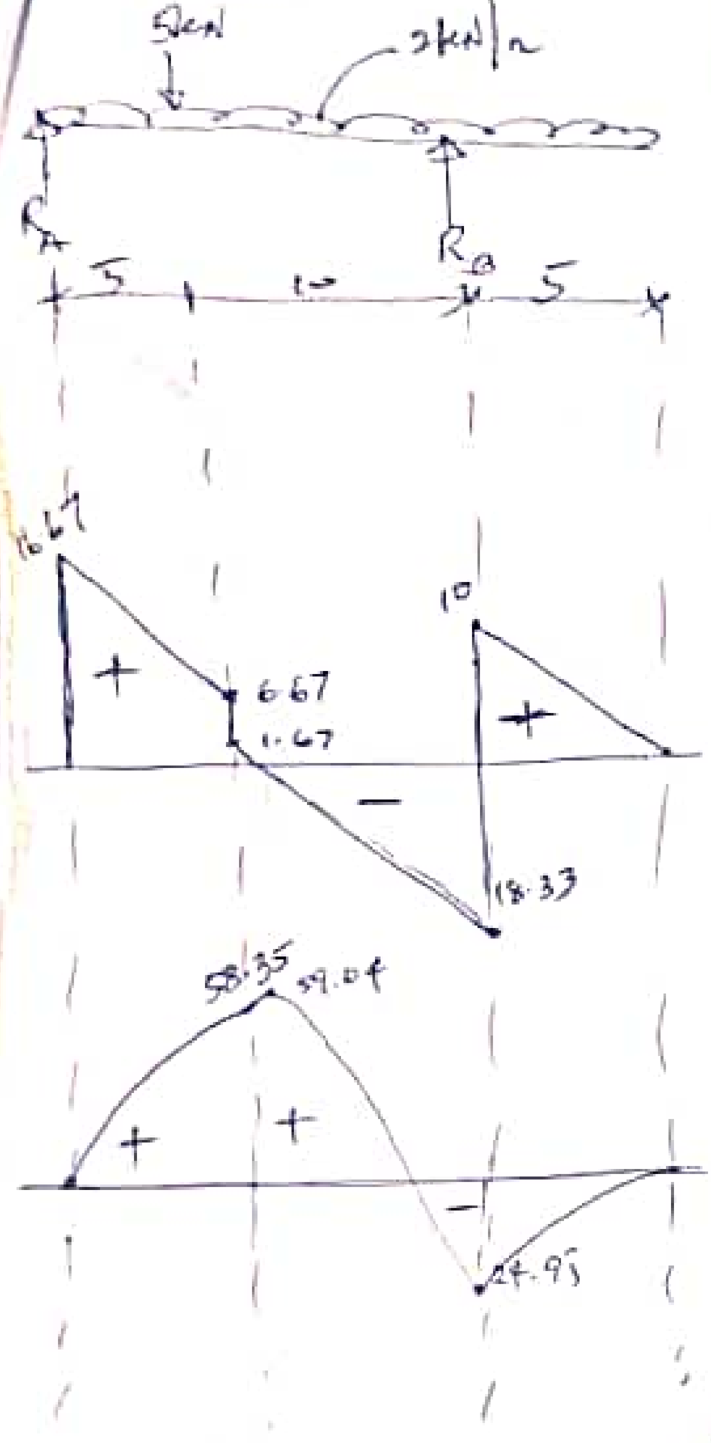


Find the reactions R_A & R_B

$R_A + R_B - 2\times20 - 5 = 0$

$R_A + R_B = 45$ ——— (1)

M_A

$-15R_B + 5\times5 + 2\times20\times\frac{20}{2} = 0$

$-15R_B + 25 + 400 = 0$

$15R_B = 425$

$R_B = 28.33KN$

from eqn 1

$R_A = 45 - R_B$

$= 45 - 28.33$

$= 16.67KN$

Sections

(1) $0 < x < 5$

(2) $5 < x < 15$

(3) $15 < x < 20$

(1) $0 < x < 5$

2kN/m $M(x)$ R_A x $F(x)$

S.F.

$R_A - 2x - F(x) = 0$

$F(x) = R_A - 2x$

$\boxed{= 16.67 - 2x}$ (2)

at $x = 0$

$F(x) = \underline{16.67KN}$

$x = 5$

$F(x) = 16.67 - 2\times5 = \underline{6.67KN}$

$M(x) = 18.33x - x^2$

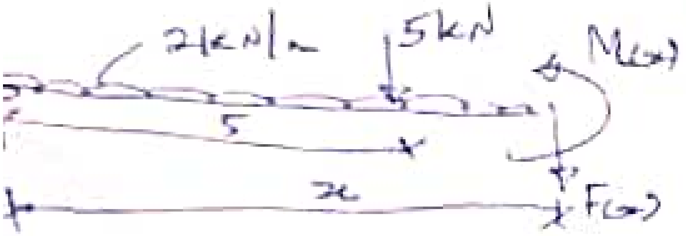
$x = 0$

$M(x=0) = 0 \text{ kN.m}$

$x = 5$

$M(x=5) = 18.33 \times 5 - (5)^2$
$= 91.65 - 25$
$= 66.65 \text{ kN.m}$

$5 \leq x \leq 15$



S.F.

$\Sigma F_y = 0$

$R_A - 2x - 5 - F(x) = 0$

$F(x) = R_A - 2x - 5$
$= 18.33 - 2x - 5$
$= 13.33 - 2x$ (*)

at $x = 5$

$F(x=5) = 13.33 - 2 \times 5$
$= 13.33 - 10$
$= 3.33 \text{ kN}$

at $x = 15$

$F(x=15) = 13.33 - 2 \times 15$
$= -16.67 \text{ kN}$

(9)

At the point where S.F crosses the axis, Σ

$F(x) = 0$

That is

$0 = 13.33 - 2x$

$x = 6.67 \text{ m from A}$

BM

$\Sigma M = 0$

$R_A x - 2 \cdot x \cdot \frac{x}{2} - 5(x-5) - M(x)$

$M(x) = R_A x - x^2 - 5x + 25$
$= 18.33x - x^2 - 5x + 25$
$= -x^2 + 13.33x + 25$

at $x = 5$

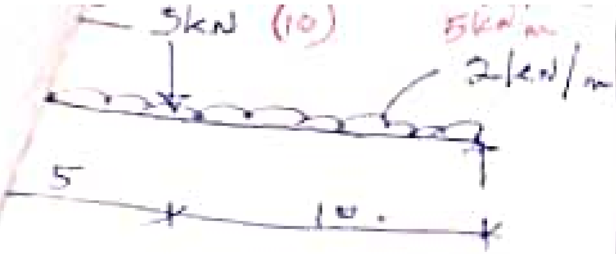
$M(x=5) = -(25) + 66.65 + 25$
$= 66.65 \text{ kN.m}$

at $x = 15$

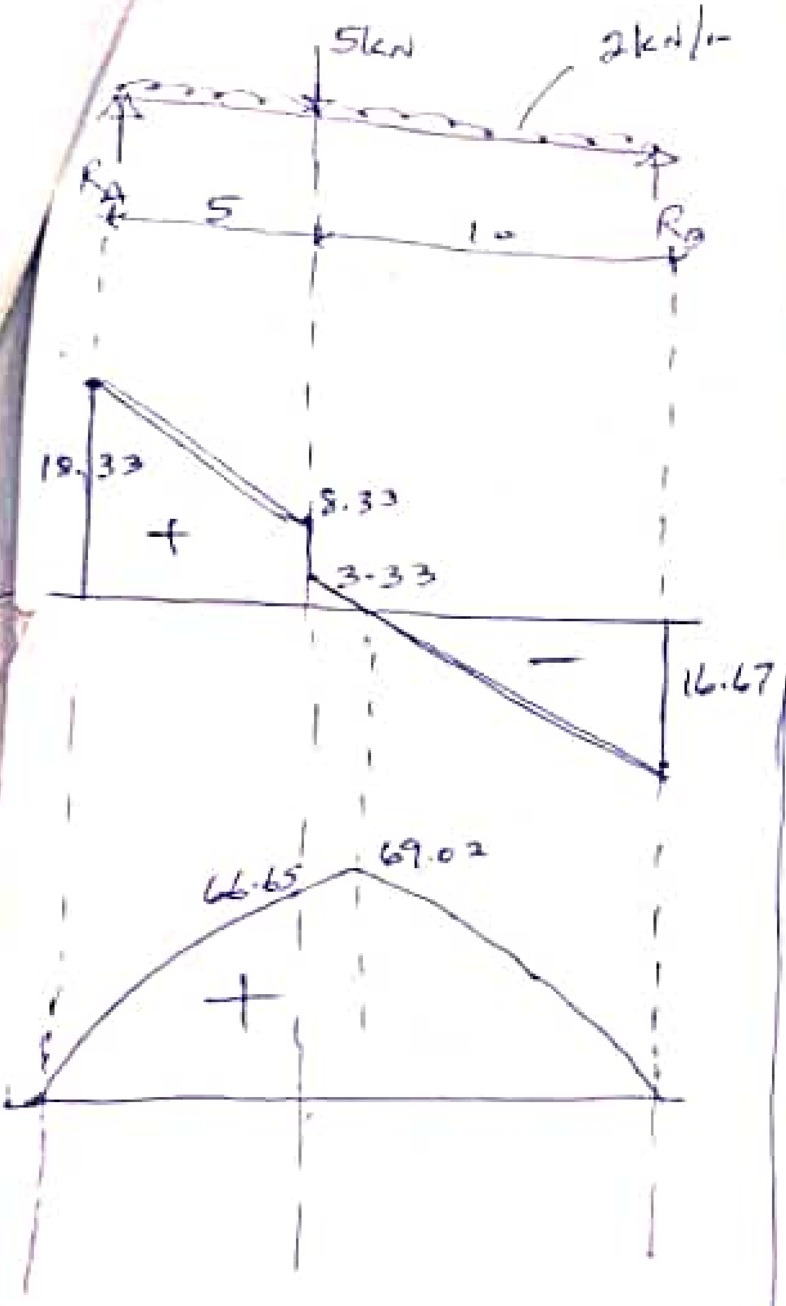
$M(x=15) = -225 + 199.95 + 25$
$= 0.05 \text{ kN.m}$
$\approx 0. \text{ kN.m}$

The max moment at $x = 6.67 \text{ m}$

$M_{max} = -(44.\overset{49}{89}) + 88.91 + 25$
$= 69.42 \text{ kN.m}$



Solution



$$R_A + R_B = 5 + 2 \times 15$$
$$= 5 + 30$$
$$= 35 \text{ kN} \quad \text{—— (1)}$$

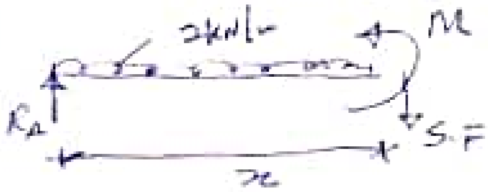
Moment about A

$$5 \times 5 + 2 \times 15 \cdot \frac{15}{2} - 15R_B = 0$$
$$15R_B = 25 + 225 = 250$$
$$\underline{R_B = 16.67 \text{ kN}}$$

from eqn 1

$$R_A = 35 - 16.67$$
$$\underline{= 18.33 \text{ kN}}$$

$$0 \leq x \leq 5$$



SF

$$\Sigma F_y = 0$$
$$R_A - 2x - F_{(x)} = 0$$
$$F_{(x)} = R_A - 2x$$
$$= 18.33 - 2x \quad (2)$$

at $x = 0$

$$F_{(x=0)} = 18.33 \text{ kN}$$

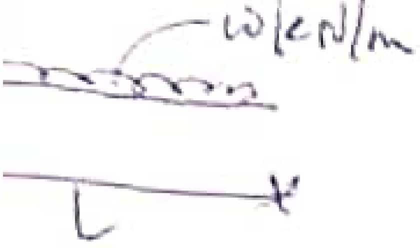
at $x = 5$

$$F_{(x=5)} = 18.33 - 2 \times 5$$
$$= 8.33 \text{ kN}$$

BM

$$R_A x - 2x \cdot \frac{x}{2} - M_{(x)} = 0$$
$$M_{(x)} = R_A x - x^2$$

ple 4



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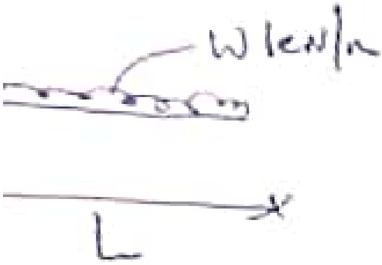
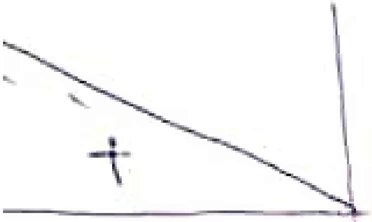
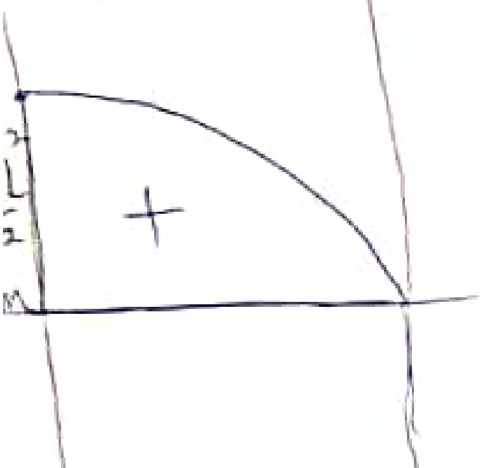
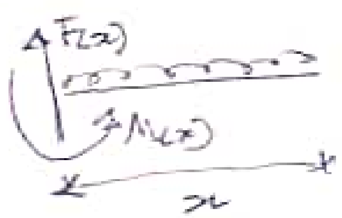






$0 < x < l$



S.F

$\sum F_y = 0$

$F(x) - wx = 0$

$F(x) = wx$ ——— (1)

That is a linear function of x

at $x = 0$

$F_{(x=0)} = w \times 0 = 0\,kN$

at $x = l$

$F_{(x=l)} = wl$

$= wl\ kN$

B.M

$\sum M_{(x)} = 0$

$-M(x) + wx \cdot \frac{x}{2} = 0$

$M(x) = \frac{wx^2}{2}$ ——— (2)

A quadratic expression

at $x = 0$

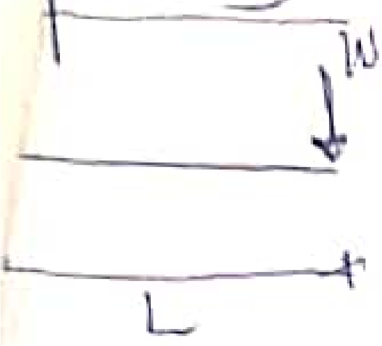
$M_{(x=0)} = 0\ kN.m$

at $x = l$

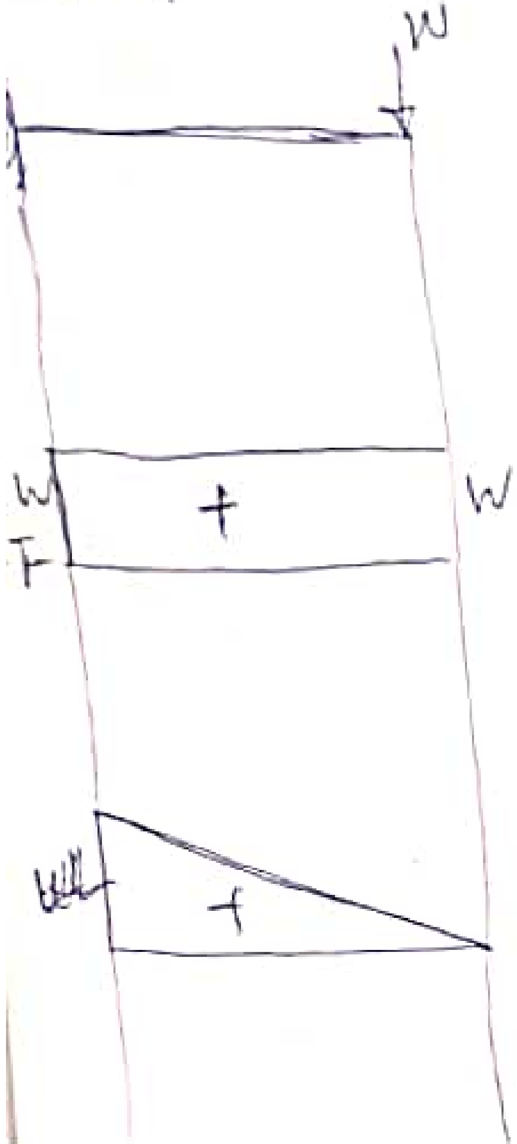
$M_{(x=l)} = \frac{wl^2}{2}$

$= \frac{wl^2}{2}\ kN.m$

Case 3

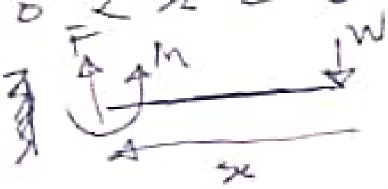


...ution



Only one section will be cut referenced at RHS.

$0 < x < l$



<u>SF</u>

$\Sigma F_y = 0$

$F - W = 0$

$F_{(x)} = W$ ——— (1)

That is, independent of the beam span. Constant through out the length

at $x = 0$

$F_{(x=0)} = W$ KN

at $x = l$

$F_{(x=l)} = W$ KN

<u>B.M</u>

$\Sigma M_x = 0$

$Wx - M_x = 0$

$M_{(x)} = Wx$ ——— (2)

A fcn of x.

at $x = 0$

$M_{(x=0)} = W \times 0$
$= 0$ KN.m

at $x = l$

$M_{(x=l)} = WL$
$= Wl$ (KN.m)

$R_A L = W ...$

The B.M at $x = \frac{L}{2}$

$$M_{(x=\frac{L}{2})} = \frac{WL}{2}\frac{L}{2} - \frac{W\left(\frac{L}{2}\right)^2}{2}$$

$$= \frac{WL^2}{4} - \frac{WL^2}{8}$$

$$= \frac{WL^2}{8} \text{ kN-m}$$